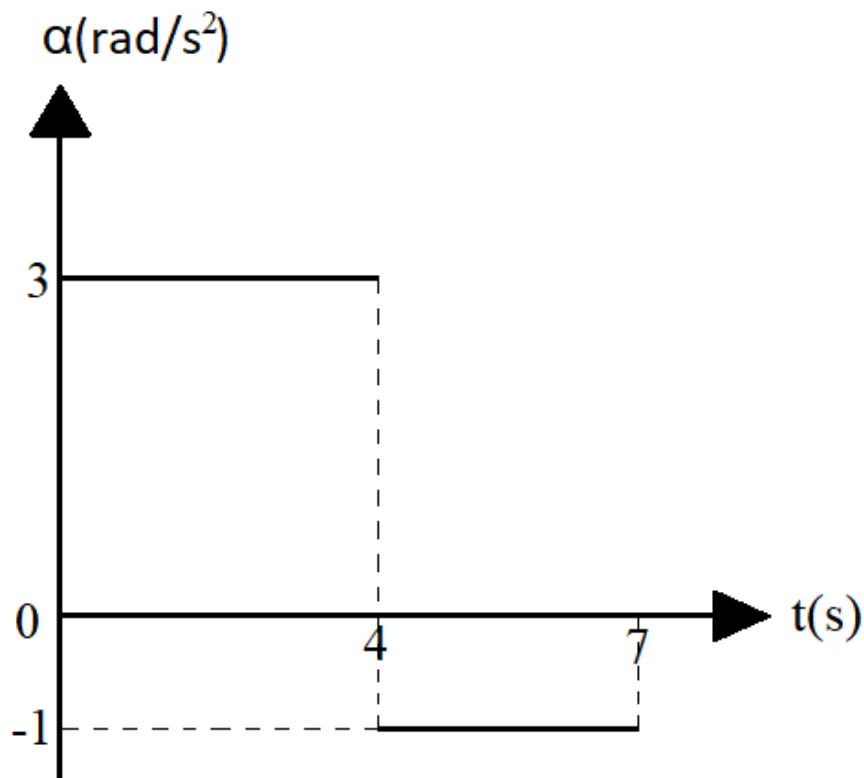


The graph of angular acceleration versus time for a rotating object is shown in the figure.

- Calculate the angular velocity and angular displacement for each interval shown in the graph.
- Calculate the total angular displacement.

Take $\varphi_0 = 0$ and $\omega_0 = 0$.



Solution

This is an example of a circular motion with constant angular acceleration. In the first part of the graph, the angular acceleration is positive. This means the object increases the angular velocity (i.e. rotates faster) by 3 rad/s in every second during the first 4 s, then it decreases the angular velocity by 1 rad/s in every second during the next 3 s. Moreover, the final angular velocity at the end of the first interval acts as an initial angular velocity for the second interval. Given this description, we have the following clues:

$$\varphi_0 = 0$$

$$\omega_0 = 0$$

$$\alpha_1 = 3 \text{ rad/s}^2$$

$$t_1 = 4 \text{ s}$$

$$\alpha_2 = -1 \text{ rad/s}^2$$

$$t_2 = 7\text{s} - 4\text{s} = 3\text{s}$$

$$\text{a) } \omega_1 = ? \qquad \omega_2 = ? \qquad \Delta\varphi_1 = ? \qquad \Delta\varphi_2 = ?$$

$$\text{b) } \Delta\varphi_{\text{total}} = ?$$

a) Using the kinematic formulas for the circular motion with constant angular acceleration we have

$$\begin{aligned}\omega_1 &= \omega_0 + \alpha_1 \cdot t_1 \\ &= 0 + \left(3 \frac{\text{rad}}{\text{s}^2}\right) \cdot (4 \text{ s}) \\ &= 12 \text{ rad/s}\end{aligned}$$

$$\begin{aligned}\omega_2 &= \omega_1 + \alpha_2 \cdot t_2 \\ &= \left(12 \frac{\text{rad}}{\text{s}}\right) + \left(-1 \frac{\text{rad}}{\text{s}^2}\right) \cdot (3 \text{ s}) \\ &= 9 \frac{\text{rad}}{\text{s}}\end{aligned}$$

$$\begin{aligned}\varphi_1 &= \varphi_0 + \omega_0 \cdot t_1 + \frac{\alpha_1 \cdot t_1^2}{2} \\ &= (0 \text{ rad}) + \left(0 \frac{\text{rad}}{\text{s}}\right) \cdot (4 \text{ s}) + \frac{\left(3 \frac{\text{rad}}{\text{s}^2}\right) \cdot (4 \text{ s})^2}{2} \\ &= 24 \text{ rad}\end{aligned}$$

$$\begin{aligned}\varphi_2 &= \varphi_1 + \omega_1 \cdot t_2 + \frac{\alpha_2 \cdot t_2^2}{2} \\ &= (24 \text{ rad}) + \left(12 \frac{\text{rad}}{\text{s}}\right) \cdot (3 \text{ s}) + \frac{\left(-1 \frac{\text{rad}}{\text{s}^2}\right) \cdot (3 \text{ s})^2}{2} \\ &= 55.5 \text{ rad}\end{aligned}$$

Therefore, the two angular displacements are

$$\Delta\varphi_1 = \varphi_1 - \varphi_0$$

$$\begin{aligned} &= 24 \text{ rad} - 0 \text{ rad} \\ &= 24 \text{ rad} \end{aligned}$$

$$\begin{aligned} \Delta\varphi_2 &= \varphi_2 - \varphi_1 \\ &= 55.5 \text{ rad} - 24 \text{ rad} \\ &= 31.5 \text{ rad} \end{aligned}$$

- b) The total angular displacement is 55.5 rad, as it represents the difference between the final and initial angular position. Thus,

$$\begin{aligned} \Delta\varphi_{total} &= \varphi_2 - \varphi_0 \\ &= 55.5 \text{ rad} - 0 \text{ rad} \\ &= 55.5 \text{ rad} \end{aligned}$$

Or

$$\begin{aligned} \Delta\varphi_{total} &= \Delta\varphi_1 + \Delta\varphi_2 \\ &= 24 \text{ rad} + 31.5 \text{ rad} \\ &= 55.5 \text{ rad} \end{aligned}$$