proprep

Workbook

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Directional Derivatives

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Questions

1) Let $f(x,y)=x^2+y^2$.

a. Compute the gradient of f and its length at the point $(3,4)$.
What is the meaning of the result?

b. Show that the gradient is normal to the contour (level curve) of f, passing through $(3,4)$.

2) Let $f(x,y)=3x^2y$. Compute the directional derivative of f at the point $(1,2)$ in the direction of the vector $\vec{u}=3\mathbf{i}+4\mathbf{j}$.

3) Let $f(x,y)=x-\sin(xy)$. Compute the directional derivative of f at the point $\left(1,\frac{\pi}{2}\right)$ in the direction of the vector $\vec{u}=\frac{1}{2}\mathbf{i}+\frac{\sqrt{3}}{2}\mathbf{j}$.

4) Let $f(x,y)=2x^2-3xy+5y^2$. Compute the directional derivative of f at the point $(1,2)$ in the direction of the unit vector which forms an angle of $45°$ with the positive x-axis.

5) Let $f(xy)=xy^2$. Compute the directional derivative of f at the point $(1,3)$ in the direction of the point $(4,5)$.

6) Let $f(x,y,z)=x^2y^2z$. Compute the directional derivative of f at the point $(2,1,4)$ in the direction of the vector $\vec{u}=1\cdot\mathbf{i}+2\cdot\mathbf{j}+2\cdot\mathbf{k}$.

7) If the electric potential V at the point (x,y) is given by $V=\ln\sqrt{x^2+y^2}$, find the rate of change of the potential at the point $(3,4)$ in the direction of the point $(2,6)$.

8) Find the direction, for which the directional derivative of the function $f(x,y)=e^x(\cos y+\sin y)$ at the point $(0,0)$ is maximal and compute its value.

9) Find the direction, for which the directional derivative of the function $f(x,y,z)=2x^3y-3y^2z$ at the point $(1,2,-1)$ is maximal and compute its value.

10) If the temperature is defined by $f(x,y,z)=3x^2-5y^2+2z^2$ and you are at the point $\left(\frac{1}{3},\frac{1}{5},\frac{1}{2}\right)$, in which direction should you go to cool off as quickly as possible?

Remark on Notation

a. In the plane R^2: $\mathbf{i}=\langle 1,0\rangle$, $\mathbf{j}=\langle 0,1\rangle$ So a vector can be denoted in two ways:
   $\vec{u}=\langle x,y\rangle$ or $\vec{u}=x\mathbf{i}+y\mathbf{j}$
   E.g., $\vec{u}=\langle 3,4\rangle \Leftrightarrow \vec{u}=3\mathbf{i}+4\mathbf{j}$
   In 3D space R^3: $\mathbf{i}=\langle 1,0,0\rangle$, $\mathbf{j}=\langle 0,1,0\rangle$, $\mathbf{k}=\langle 0,0,1\rangle$
   So a vector can be denoted in two ways: $\vec{v}=\langle x,y,z\rangle$ or $\vec{v}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$
   E.g., $\vec{u}=\langle 3,4,5\rangle \Leftrightarrow \vec{u}=3\cdot\mathbf{i}+4\cdot\mathbf{j}+5\cdot\mathbf{k}$

b. Elsewhere, a vector $\vec{u}$ may be denoted as $\underline{u}$ or as $\mathbf{u}$.

c. A unit vector will be denoted $\hat{\mathbf{u}}$.

Answer Key

1) The gradient is $\langle 6,8\rangle$ and its length is 10.

2) $\frac{5}{48}$

3) $\frac{1}{2}$

4) $7.5\sqrt{2}$

5) $3\sqrt{13}$

6) $\frac{3}{88}$

7) $\frac{1}{5\sqrt{5}}$

8) It is maximal the in the direction of the vector $\langle 1,1\rangle$ and is equal to $\sqrt{2}$.

9) It is maximal the in the direction of the vector $\langle 12,14,-12\rangle$ and is equal to 22.

10) In the direction of the vector $\langle -2,2,-2\rangle$.