proprep™

Workbook

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Early Transcendentals – 14th Edition

Vectors and the Geometry of Space

Three-Dimensional Coordinate Systems

Questions

1) Find the projection of the point $\mathrm{P}(4,7,-5)$ onto the three coordinate planes.

2) Answer the following questions:

a. What is the distance of the point $\mathrm{P}(4,7,-5)$ from the z-axis?

b. Which point is closer to the z-axis: $\mathrm{P}(4,7,-5)$ or $\mathrm{Q}(5,-6,7)$?

3) Answer the following questions:

a. What is the distance of the point $\mathrm{P}(4,7,-5)$ from the xy-plane?

b. Which point is closer to the xy-plane: $\mathrm{P}(4,7,-5)$ or $\mathrm{Q}(5,-6,7)$?

4) Answer the following questions:

a. Given the equation $2x+3y=6$,

i. In $\mathbb{R}^2$ this is the equation of a:_____

ii. In $\mathbb{R}^3$ this is the equation of a:_____

b. Given the equation $(x-1)^2+y^2=9$,

i. In $\mathbb{R}^2$ this is the equation of a: _____

ii. In $\mathbb{R}^3$ this is the equation of a: _____

Answer Key

1) Onto xy: $(4,7,0)$ Onto xz: $(4,0,-5)$ Onto yz: $(0,7,-5)$

2) a. 5 b. Point P is closer.

3) a. $\sqrt{65}$ b. Point Q is closer.

4) a.(i) Line. (ii) Plane. b.(i) Circle. (ii) Cylinder.

Vectors

Questions

1) Find each of the following 2D/3D vectors, its magnitude, and whether it's a unit vector:

a. The displacement vector from $(-8,3)$ to $(5,-2)$.

b. The displacement vector from $(2,3,4)$ to $(2,4,4)$.

c. The position vector for $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$.

d. The position vector for $(-8,3,5)$.

e. The vector $\vec{v}=\langle 7,-3,0\rangle$ starts at point $\mathrm{P}(-3,4,-1)$.
At what point does it end?

Vector Arithmetic

2) Given the 2D vectors $\vec{a}=\langle 7,4\rangle$ and $\vec{b}=\langle -2,5\rangle$, compute the following:

a. $5\vec{a}$

b. $6\vec{b}-3\vec{a}$

c. $\left\|9\vec{a}+4\vec{b}\right\|$

3) Given the 3D vectors $\vec{u}=7\vec{i}-2\vec{j}+4\vec{k}$ and $\vec{v}=6\vec{j}-2\vec{k}$, compute the following:

a. $-4\vec{v}$

b. $10\vec{u}+\vec{v}$

c. $\|-8\vec{u}-3\vec{v}\|$

4) Answer the following questions:

a. Find a unit vector that points in the same direction as $\vec{v}=\vec{i}-4\vec{j}+8\vec{k}$.

b. Find a vector that points in the same direction as $\vec{w}=\langle -2,5\rangle$ with a magnitude of 10.

5) Determine if the following pairs of vectors are parallel.

a. $\vec{v}=6\vec{i}-4\vec{j}-16\vec{k}$ and $\vec{w}=15\vec{i}-10\vec{j}-40\vec{k}$.

b. $\vec{a}=\langle 3,-2,5\rangle$ and $\vec{b}=\langle 6,-4,7\rangle$.

Answer Key

1) a. $\langle 13,-5\rangle$ b. $\langle 0,1,0\rangle$ c. $\left\langle \frac{1}{2},\frac{\sqrt{3}}{2}\right\rangle$ d. $\langle -8,3,5\rangle$; $(4,1,-1)$

2) a. $\langle 35,20\rangle$ b. $\langle -33,18\rangle$ c. $\sqrt{6161}$

3) a. $-24\vec{j}+8\vec{k}$ b. $70\vec{i}-14\vec{j}+38\vec{k}$ c. $\sqrt{3816}$

4) a. $\frac{1}{9}\vec{v}-\frac{4}{9}\vec{j}+\frac{8}{9}\vec{k}$ b. $\left\langle \frac{-20}{\sqrt{29}},\frac{50}{\sqrt{29}}\right\rangle$

5) a. Parallel b. Not parallel

The Dot Product

Questions

1) In each of the following, find the dot product $\vec{a}\cdot\vec{b}$:

a. $\vec{a}=\langle 5,-4\rangle$ and $\vec{b}=\langle 4,3\rangle$.

b. $\vec{a}=8\vec{i}+6\vec{j}-3\vec{k}$ and $\vec{b}=6\vec{i}-4\vec{j}+7\vec{k}$.

c. $\|\vec{a}\|=4$, $\|\vec{b}\|=3$, and the angle between the two vectors is $\frac{\pi}{3}$.

2) In each of the following, find the angle between the two vectors:

a. $\vec{a}=\langle 3,5\rangle$, $\vec{b}=\langle 7,6\rangle$.

b. $\vec{v}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{w}=5\vec{i}+6\vec{j}-7\vec{k}$.

3) In each of the following, determine if the two vectors are parallel, orthogonal or neither:

a. $\vec{p}=\langle 1,-2,3\rangle$, $\vec{q}=\langle 5,-8,-7\rangle$.

b. $\vec{a}=\langle 3,5\rangle$, $\vec{b}=\langle 7,6\rangle$.

c. $\vec{v}=\vec{i}-2\vec{j}+3\vec{k}$, $\vec{w}=-5\vec{i}+10\vec{j}-15\vec{k}$.

4) Compute $proj_{\vec{u}}\vec{v}$ [projection of $\vec{v}$ onto $\vec{u}$] for the following pairs of 2D/3D vectors:

a. $\vec{u}=\langle 4,-1\rangle$, $\vec{v}=\langle 1,7\rangle$.

b. $\vec{u}=7\vec{i}-\vec{j}+\vec{k}$, $\vec{v}=-2\vec{i}+5\vec{j}-6\vec{k}$.

5) Find the direction cosines and direction angles for $\vec{v}=\vec{i}-2\vec{j}+3\vec{k}$.

Answer Key

6) a. 8 b. 3 c. 6

7) a. $18.435°$ b. $44.479°$

8) a. orthogonal b. neither c. parallel

9) a. $\left\langle \frac{-12}{17},\frac{3}{17}\right\rangle$ b. $\left\langle \frac{-161}{51},\frac{23}{51},\frac{-23}{51}\right\rangle$

10) $\sqrt{14}$

The Cross Product

Questions

1) Answer the following questions:

a. Given $\vec{a}=\langle 3,-2,5\rangle$ and $\vec{b}=\langle 6,-4,7\rangle$.

Compute $\vec{a}\times\vec{b}$ and use the result to find $\vec{b}\times\vec{a}$.

b. Given $\vec{u}=3\vec{i}-\vec{j}+5\vec{k}$ and $\vec{v}=4\vec{j}-2\vec{k}$.

Compute $\vec{u}\times\vec{v}$ and use the result to find $\vec{v}\times\vec{u}$.

2) Find a vector that is orthogonal to the plane containing the points $\mathrm{P}(1,2,3)$, $\mathrm{Q}(6,5,4)$ and $\mathrm{R}(7,8,9)$.

3) For each of the following sets of 3 vectors, determine if they lie in the same plane or not:

a. $\vec{a}=\langle 3,-2,5\rangle$, $\vec{b}=\langle 6,-4,7\rangle$, $\vec{c}=\langle 1,0,1\rangle$.

b. $\vec{u}=\vec{i}+4\vec{j}-7\vec{k}$, $\vec{v}=2\vec{i}-\vec{j}+4\vec{k}$, $\vec{w}=-9\vec{j}+18\vec{k}$.

Answer Key

1) Part 1: $\langle 6,9,0\rangle$ Part 2: $18\vec{i}-6\vec{j}+12\vec{k}$

2) $\langle 12,-24,12\rangle$

3) a. Not in the same plane b. In the same plane.

Lines and Planes in Space

Questions

Equations of Lines

1) Find the three forms [vector, parametric, symmetric] of the equation of the line which passes through the points $(-10,4,0)$ and $(1,-4,2)$.

2) Find the three forms [vector, parametric, symmetric] of the equation of the line which passes through the point $(-10,4,0)$ and is parallel to the line $x=3+4t$, $y=-2+3t$, $z=-5t$.

3) Let l_1 be the line through points $(4,1,-5)$ and $(2,0,9)$ and let l_2 be the line given by $\vec{r}(t)=\langle 5,1-9t,-8-4t\rangle$. Are the lines l_1 and l_2 parallel, perpendicular or neither?

4) Let l_1 be the line given by $x=-7+12t$, $y=3-t$, $z=14+8t$ and let l_2 be the line given by $\vec{r}(t)=\langle 8+t,5+6t,4-2t\rangle$. Do l_1 and l_2 intersect? If so, find the intersection point.

5) Let l_1 be the line passing through points $(-5,0,2)$ and $(13,-2,1)$ and let l_2 be the line given by $\vec{r}(t)=\langle 3,-1-t,2+4t\rangle$. Do l_1 and l_2 intersect? If so, find the intersection point.

6) Let l be the line given by $x=-7+12t$, $y=3$, $z=16+8t$.

a. Does l intersect the xy-plane? If so, where?

b. Does l intersect the xz-plane? If so, where?

Equations of Planes

Questions

7) Find the equation of the plane through the points $\mathrm{P}(0,1,1)$, $\mathrm{Q}(1,0,1)$ and $\mathrm{R}(1,-3,-1)$.

8) Find the equation of the plane passing through the point $(0,2,-1)$ and orthogonal to the line $\vec{r}(t)=\langle 5+t,1+3t,4t\rangle$.

9) Find the equation of the plane containing the point $(-7,3,9)$ and parallel to the plane $4x+8y-2z=37$.

10) Plane π_1 is given by $4x+8y-2z=10$ and plane π_2 is given by $2x+y+8z=11$. Are the planes π_1 and π_2 parallel, orthogonal or neither?

11) Plane π_1 is given by $2x-3y+4z=5$ and plane π_2 passes though points $(1,2,2)$, $(2,2,3)$ and $(-3,-2,-6)$. Are the planes π_1 and π_2 parallel, orthogonal or neither?

12) Plane π is given by $2x-y+3z=6$ and line l is given by $x=1-t$, $y=3t$, $z=1+t$.
Do l and π intersect? If so, where?

13) Plane π is given by $x-y+z=3$ and line *l* is given by $\vec{r}(t)=\langle 5+2t,1-5t,3t\rangle$.
Do l and π intersect? If so, where?

14) Two planes π_1 and π_2 are given by $-x+7y-2z=24$ and $-5x+6y+3z=-3$, respectively. l is the line intersection of the planes: $l=\pi_1\cap\pi_2$. Find the vector equation of line l.

15) Plane π is given by $5x-3y-6z=4$ and line l is given by $\vec{r}(t)=\langle 5-10t,1+6t,12t\rangle$.
Are l and π parallel, perpendicular or neither?

Answer Key

1) $\langle -10+11t, 4-8t, 2t \rangle$ $x=-10+11t$, $y=4-8t$, $z=-2t$ $\frac{x+10}{11}-\frac{y-4}{8}=\frac{z}{2}$

2) $\langle -10+4t, 4+3t, -5t \rangle$ $t=\frac{x+10}{4}$, $t=\frac{y-4}{3}$, $t=-\frac{z}{5}$ $\frac{x+10}{4}=\frac{y-4}{3}=-\frac{z}{5}$

3) Perpendicular.

4) $v=\frac{-39}{73}$; $u=\frac{88}{73}$

5) $\left(3,-\frac{8}{9},1\frac{5}{9}\right)$

6) a. $(-31,3,0)$ b. Not intersection.

7) $2x+2y-3z=-1$

8) $x+3y+4z=2$

9) $4x+8y-2z=-22$

10) Orthogonal.

11) Neither.

12) $\left(1\frac{1}{2},-1\frac{1}{2},\frac{1}{2}\right)$

13) Not intersect.

14) $\vec{r}(t)=\langle 33t, 2+13t, -5+29t \rangle$

15) Perpendicular.

Cylinders and Quadric Surfaces

Questions

1) Sketch the graph of the quadric surface $\frac{y^2}{9}+z^2=1$.

2) Sketch the graph of the quadric surface $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{6}=1$.

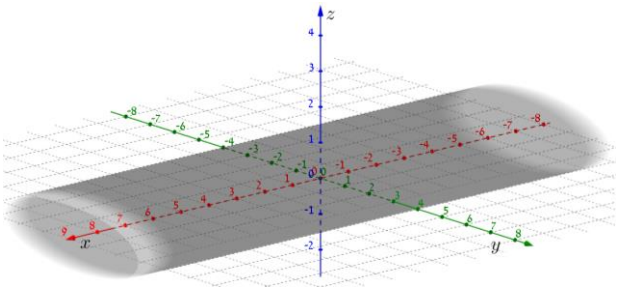
3) Sketch the graph of the quadric surface $z=\frac{x^2}{4}+\frac{y^2}{4}-6$.

4) Sketch the graph of the quadric surface $y^2=4x^2+16z^2$.

5) Sketch the graph of the quadric surface $x=4-5y^2-9z^2$.

Answer Key

1)



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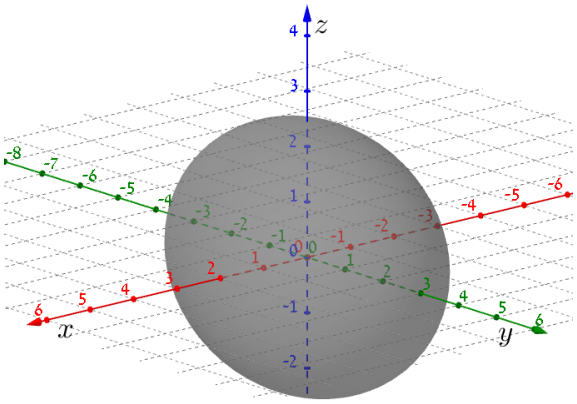

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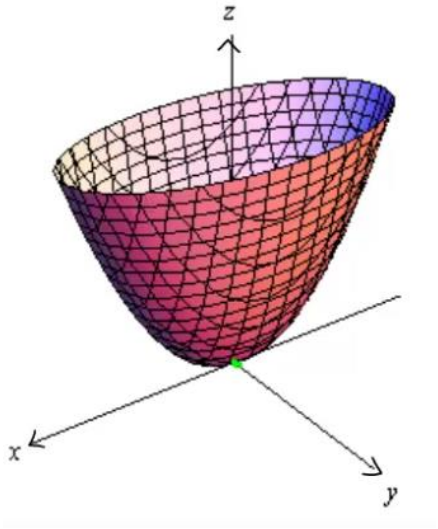

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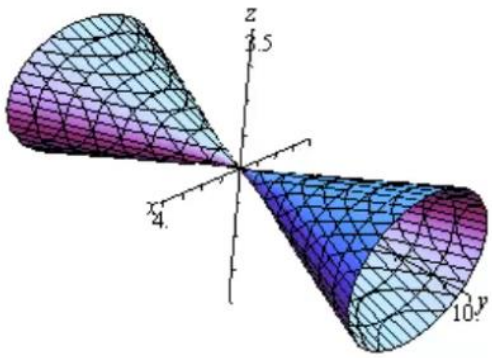

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